

$B \rightarrow X_d \gamma$ and constraints on new physics

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We combine recent progress in measuring the branching ratio of the decay $B \rightarrow X_d \gamma$ with the discovery that hadronic uncertainties in the CP-averaged branching ratio drop out to a large extent. Implications of these improvements on the size of possible new physics effects are investigated. We find the updated SM prediction for the CP-averaged branching ratio to be $\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} \rangle = 1.54_{-0.31}^{+0.26} \times 10^{-5}$, which should be compared with the experimental value of $\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} \rangle = (1.41 \pm 0.57) \times 10^{-5}$. After performing a model independent analysis, we consider different new physics models: the MSSM with generic sources of flavor violation, the two Higgs doublet model of type III and a model with right-handed charged currents. It is found that the constraints on the SUSY parameters δ_{13}^d have improved and that the absolute value of the right-handed quark mixing matrix element $|V_{td}^R|$ must be smaller than 1.5×10^{-4} .

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I. INTRODUCTION

In the past, the main focus has been on the inclusive decay $B \rightarrow X_s \gamma$ while its analog with a down quark in the final state, $B \rightarrow X_d \gamma$, received much less attention. The reason for this was that both the experimental measurement $\text{Br}[B \rightarrow X_s \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} = (3.60 \pm 0.23) \times 10^{-4}$ [1] and the standard model prediction (it is now known to NNLO precision) $\text{Br}[B \rightarrow X_s \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$ [2, 3] of this decay were significantly better compared to $B \rightarrow X_d \gamma$. However, this situation has changed recently:

- The new CP-averaged branching ratio $\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} \rangle = (1.41 \pm 0.57) \times 10^{-5}$ of the BABAR collaboration [4, 5] (CP averaging is denoted by $\langle \dots \rangle$ throughout this article) is more precise than the previous one and the photon cut is lower which reduces the error of the extrapolation to 1.6 GeV¹. Furthermore, there are good experimental prospects for this decay: the analysis of existing BELLE data and the future super-B factories [6, 7] will allow for a more precise determination of this branching ratio.
- The theory prediction for the standard model (SM) contribution has been calculated in Ref. [8] and the NLO QCD corrections can be found in Ref. [9]. As

in the case of $B \rightarrow X_s \gamma$, also $B \rightarrow X_d \gamma$ suffers from hadronic uncertainties but for the latter the non-perturbative contributions from up-quark loops are not CKM-suppressed which magnifies the error of the theory prediction. However, it has been only recently realized that most of these uncertainties drop out in the CP-averaged branching ratio [10, 11]. Thus, the SM prediction for $B \rightarrow X_d \gamma$ can in principle be calculated with the same accuracy as $B \rightarrow X_s \gamma$.

- In addition, the error in the determination of the CKM element V_{td} has constantly decreased in the last years [12, 13]. This further reduces the uncertainty of the SM contribution to $B \rightarrow X_d \gamma$ which depends quadratically on V_{td} . The uncertainty coming from the determination of V_{td} now only induces an error in the SM branching ratio of approximately 10% if one varies the value of V_{td} within its 95%CL region.

These significant improvements and promising prospects on the theoretical as well as on the experimental side motivate us to perform an updated analysis of $B \rightarrow X_d \gamma$ and the constraints placed from this decay.

II. EFFECTIVE HAMILTONIAN AND SM PREDICTION

In the SM the effective Hamiltonian governing $\bar{B} \rightarrow X_d \gamma$ is given by:

$$\mathcal{H}_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} V_{td}^* V_{tb} \left[\sum_{i=1}^8 C_i O_i + \epsilon_d \sum_{i=1}^2 C_i (O_i - O_i^u) \right], \quad (1)$$

where $O_1^u, O_2^u, O_1, \dots, O_6$ are four-quark operators, $\epsilon_d = \frac{V_{ud}^* V_{ub}}{V_{td}^* V_{tb}}$ and the (chromo)magnetic operator (O_8) O_7 is

¹ Note that in the ICHEP 2010 update of HFAG the value for $B \rightarrow X_d \gamma$ is not extrapolated from the photon cut of 2.26 GeV used in the BABAR measurement to a cut of 1.6 GeV even though this was done in previous updates. It can thus be misleading to compare the value of HFAG with the one of PDG (quoted in the HFAG analysis) since the latter one has been extrapolated down to 1.6 GeV. We obtained the value quoted above by using the extrapolation of HFAG. In order to be conservative we doubled the error given in www.slac.stanford.edu/xorg/hfag/rare/ichep10/radll/btosg.pdf.

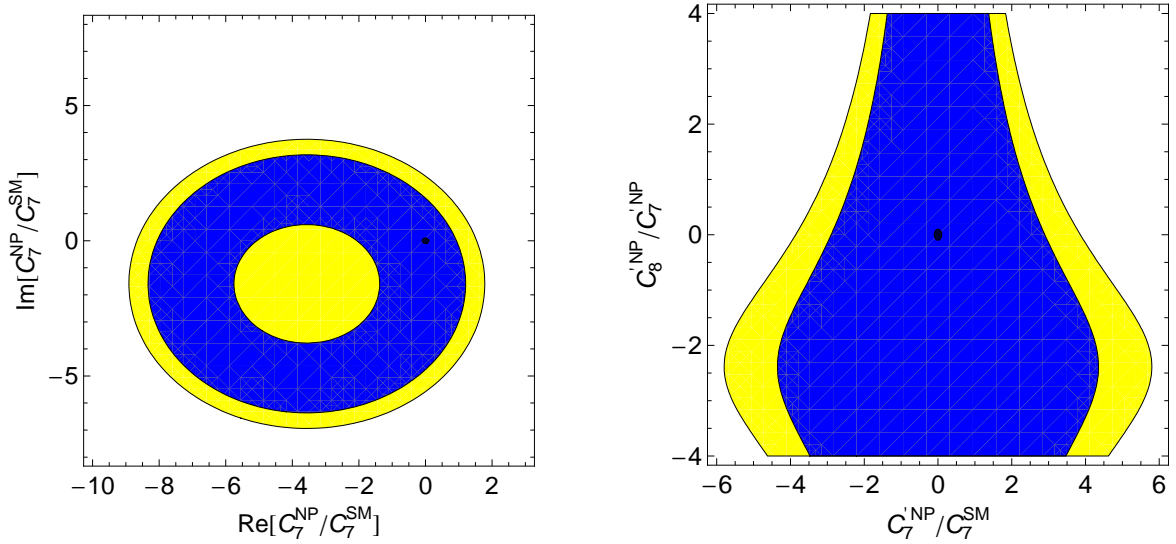


FIG. 1: The blue (yellow) region agrees with the measured branching ratio at the 1σ (2σ) level. Left plot: Allowed region in the $\text{Re}[C_7^{\text{NP}}/C_7^{\text{SM}}] - \text{Im}[C_7^{\text{NP}}/C_7^{\text{SM}}]$ plane for $C_7^{\text{NP}}/C_8^{\text{NP}} = C_7^{\text{SM}}/C_8^{\text{SM}}$ and $C_7^{\text{NP}} = 0$. Right plot: Allowed region in the $C_7^{\text{NP}}/C_7^{\text{SM}} - C_8^{\text{NP}}/C_7^{\text{NP}}$ plane for $C_8^{\text{NP}}/C_7^{\text{NP}} \in \mathbb{R}$. Note that the constraints in the right plot are independent of the phase of C_7^{NP} . In both plots the SM point is marked by a black dot.

given by

$$\begin{aligned} O_7 &= \frac{e}{16\pi^2} m_b(\mu) (\bar{d}_L \sigma_{\mu\nu} b_R) F^{\mu\nu}, \\ O_8 &= \frac{g_s}{16\pi^2} m_b(\mu) (\bar{d}_L T^a \sigma_{\mu\nu} b_R) G^{a\mu\nu}. \end{aligned} \quad (2)$$

In the presence of new physics (NP), additional operators may appear. We assume that the only sizeable NP contributions enter through $O_{7,8}^{(\prime)}$ which is the case for the models under consideration in this article. The operators $O_{7,8}'$ are obtained by exchanging L with R and vice versa in the unprimed operators. The NLO decay width can thus be written as

$$\text{Br}[\bar{B} \rightarrow X_d \gamma] = \mathcal{N} \left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 (P + N + P'). \quad (3)$$

Here P contains the perturbative SM contributions and the NP contributions to $O_{7,8}$ while P' contains only the NP contributions to $O_{7,8}'$. N denotes the non-perturbative corrections and $\mathcal{N} \approx 2.5 \times 10^{-3}$ is a numerical prefactor (see Ref. [14] for details).

Before turning our attention to NP, we update the SM prediction for the CP averaged branching ratio $\langle \text{Br}[B \rightarrow X_d \gamma]_{\text{SM}} \rangle$ of Refs. [9, 14] by using the improved determination of the CKM-element V_{td} and the reduced non-perturbative uncertainties which are estimated to be at most 5% (as for $B \rightarrow X_s \gamma$) [10, 11]. The remaining leading uncertainty stems from renormalization scheme dependence of the ratio m_c/m_b (approximately 15%) which is supplemented by a 3.5% scale ambiguity and a 6% parametric uncertainty [14]. In addition there is still a 10% change in the branching ratio if one varies

V_{td} within its 95%CL region. Adding all these uncertainties in quadrature, we get:

$$\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} \rangle = 1.54_{-0.31}^{+0.26} \times 10^{-5}. \quad (4)$$

Comparing this with the experimental value of $\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} \rangle = (1.41 \pm 0.57) \times 10^{-5}$, we see that the SM prediction well within the experimental 1σ range.

III. CONSTRAINTS ON NEW PHYSICS

Since the SM contribution to $B \rightarrow X_d \gamma$ is not only loop- but also chirality-suppressed, this decay is, just as $B \rightarrow X_s \gamma$, very sensitive to new sources of flavor and chirality violation which occur in most NP models. In order to include NP into the calculation of the branching ratio we rely on the NLO formula of Ref. [14].

The constraints in this section are obtained by demanding that branching ratio, including NP contributions, should lie within the 2σ range of the experimental values if not indicated otherwise. In order to give a conservative estimate we add the theory error and the experimental one linearly. Further we define:

$$C_{7,8}^{\text{NP}} = C_{7,8} - C_{7,8}^{\text{SM}}, \quad C_{7,8}'^{\text{NP}} = C_{7,8}'. \quad (5)$$

A. Model independent analysis

Firstly, we can constrain the Wilson coefficients $C_7^{\text{NP}(\prime)}$ and $C_8^{\text{NP}(\prime)}$ at the scale M_W . In the left plot of

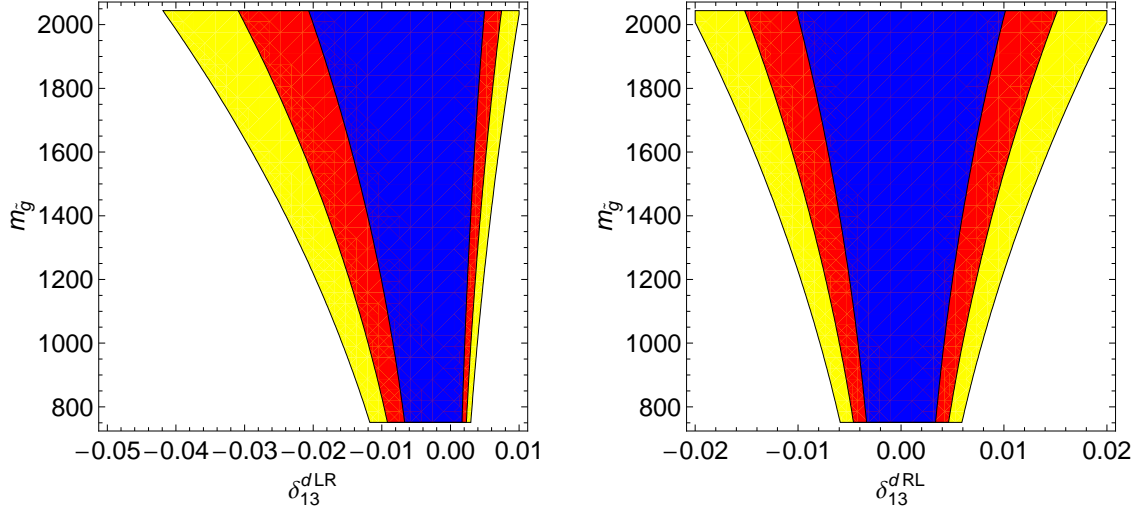


FIG. 2: Constraints on the mass insertion parameters δ_{13}^{dLR} (left plot) and δ_{13}^{dRL} (right plot) for $m_{\tilde{g}} = m_{\tilde{q}} = 1$ TeV and $\mu \tan \beta = 30$ TeV (yellow), $\mu \tan \beta = 0$ (red), $\mu \tan \beta = -30$ TeV (blue). Note that we only considered the leading gluino contributions (see Ref. [15] for details). The constraints on δ_{13}^{dRL} are independent of its phase while the constraints on δ_{13}^{dLR} are given for $\text{Arg}[\delta_{13}^{dRL}] = \text{Arg}[V_{td}]$ and have to be scaled according to Fig. 1 otherwise. In order to take into account the chirally enhanced corrections we used the effective Feynman rules of Ref. [16].

Fig. 1 we show the 1σ and 2σ allowed region in the $\text{Re}[C_7^{\text{NP}}/C_7^{\text{SM}}] - \text{Im}[C_7^{\text{NP}}/C_7^{\text{SM}}]$ plane for $C_7^{\text{NP}}/C_8^{\text{NP}} = C_7^{\text{SM}}/C_8^{\text{SM}}$. Clearly the size of constructive contributions is very limited, but large destructive contributions are still possible. The primed operators always give a constructive contribution to the branching ratio and thus their possible size is rather limited (see right plot of Fig. 1). Note that $|C_{7,8}^{\text{NP}}|$ can easily be several times larger than $|C_{7,8}^{\text{SM}}|$ in the case of destructive interference. The reason for this is that we normalize $C_{7,8}^{\text{NP}}$ only to $C_{7,8}^{\text{SM}}$, i.e. even for $C_{7,8}^{\text{NP}} = -C_{7,8}^{\text{SM}}$ the branching ratio is not zero because of the contributions from the SM four-quark operators.

In models with minimal flavor violation (MFV) [17], the constraints on the Wilson coefficients $C_7^{\text{NP}(\prime)}$ and $C_8^{\text{NP}(\prime)}$ obtained in this section can be directly compared to the ones from $b \rightarrow s\gamma$ because the CKM elements are factored out in Eq. (1). Note that despite the recent improvements in $b \rightarrow d\gamma$ the constraints from $b \rightarrow s\gamma$ are still stronger if MFV is assumed.

B. MSSM

The generic MSSM possesses many new sources of flavor violation and constraining this flavor structure with FCNC processes has a long and fruitful tradition [18, 19]. Concerning $B \rightarrow X_d\gamma$, we are especially sensitive to the chirality flipping elements $\delta_{13}^{dLR,RL}$ [20] (but also to $\delta_{13}^{dLL,RR}$ at moderate to large values of $\tan \beta$) and we get even more stringent constraints than from $B_d - \bar{B}_d$ mixing [21]. The results are depicted in Fig. 2.

C. 2HDM of type III

Despite the significant improvements in $B \rightarrow X_d\gamma$, the bounds from $B \rightarrow X_s\gamma$ are still tighter in scenarios with MFV [17]. Thus the constraints on the charged Higgs mass of a two Higgs doublet models (2HDM) of type II are still more stringent from $B \rightarrow X_s\gamma$. However in a 2HDM of type III the non-holomorphic couplings of a t and a u quark to the Higgs can be constrained. This kind of models have been considered in Refs. [22, 23, 24], where however additional assumptions on the structure of the couplings has been imposed for the phenomenological studies.

Following the notation of Ref. [25] we denote the coupling coefficients of the charged Higgs vertex (for large $\tan \beta$) as

$$\begin{aligned} \Gamma_{u_f d_i}^{LR H^\pm} &= \frac{1}{v} \sum_{j=1}^3 V_{fj}^{CKM} \tan(\beta) \left(m_{d_i} \delta_{ji} - \tilde{\Sigma}_{ji}^{dLR} \right) \\ \Gamma_{u_f d_i}^{RL H^\pm} &= \frac{1}{v} \sum_{j=1}^3 \left(\tan(\beta) \tilde{\Sigma}_{fj}^{uRL} \right. \\ &\quad \left. + \cot(\beta) m_{u_f} \delta_{fj} \right) V_{ji}^{CKM}. \end{aligned} \quad (6)$$

In Fig. 4 we show the constraints that we get on the product $\tan(\beta)^2 \tilde{\Sigma}_{31}^{uRL}$ which therefore, up to strongly suppressed terms, depend only on the charged Higgs mass. In principle we could also consider $\tilde{\Sigma}_{ji}^{dLR}$, but the bounds from $B_d \rightarrow \mu^+ \mu^-$ are more stringent.

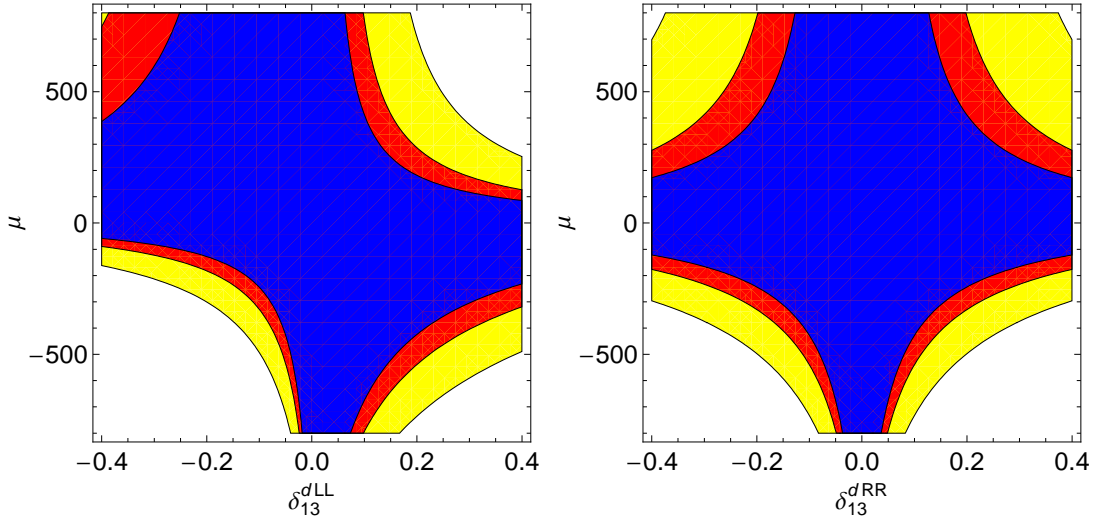


FIG. 3: Constraints on the mass insertion parameters δ_{13}^{dLR} (left plot) and δ_{13}^{dRL} (right plot) for $\tan\beta = 50$, $m_{\tilde{g}} = 1$ TeV and $m_{\tilde{g}} = 1.5$ TeV (yellow), $m_{\tilde{g}} = 1$ TeV (red) $m_{\tilde{g}} = 0.75$ TeV (blue). The constraints on δ_{13}^{dLL} are independent of its phase while the constraints on δ_{13}^{dRR} are given for $\text{Arg}[\delta_{13}^{dRR}] = \text{Arg}[V_{td}]$ and have to be scaled according to Fig. 1 if the phase is different.

D. Right-handed charged currents

It is well known that $B \rightarrow X_s \gamma$ puts stringent constraints on models with right-handed charged currents [26, 27]. We can thus also constrain the elements of the right-handed mixing matrix through $B \rightarrow X_d \gamma$ ². We define the effective W -quark-quark vertex as

$$i\Gamma_{t,d}^{W\mu} = -i\frac{g_2}{\sqrt{2}}\gamma^\mu (V_{td}P_L + V_{td}^R P_R) . \quad (7)$$

If $V_{td}^R \neq 0$ contributions to $C'_{7,8}$ are induced which necessarily enhance the branching ratio. Using the formulas of Ref. [27] and assuming the SM value for V_{td} , we get the following limit on V_{td}^R

$$|V_{td}^R| \leq 1.5 \times 10^{-4} . \quad (8)$$

Note that this constraint is approximately 3.5 times stronger than what is found for the best-fit solution of the right-handed CKM matrix in Ref. [29].

IV. CONCLUSIONS AND OUTLOOK

In this letter we studied the constraints on NP from the inclusive radiative decay $B \rightarrow X_d \gamma$. Including the improved determination of V_{td} and the reduced hadronic uncertainties [10, 11] in the CP-averaged branching ratio, the new NLO SM prediction is given by $\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{SM}} \rangle = 1.54_{-0.31}^{+0.26} \times 10^{-5}$. If we extrapolate the experimental value from the BABAR collaboration [4] to a photon energy cut of 1.6 GeV, we get $\langle \text{Br}[B \rightarrow X_d \gamma]_{E_\gamma > 1.6 \text{ GeV}}^{\text{exp}} \rangle = (1.41 \pm 0.57) \times 10^{-5}$.

We found constraints on the parameters $\delta_{13}^{dLR,RL}$ of the MSSM squark mass matrices which are more stringent than the ones obtained from $B-\bar{B}$ mixing. Also, an effective right-handed W coupling to the top and down quarks is severely constrained: $|V_{td}^R| \leq 1.5 \times 10^{-4}$. This for example strongly disfavors the proposed best-fit solution to the right-handed CKM matrix of Ref. [29].

The significance of $B \rightarrow X_d \gamma$ can even be improved by a NNLO computation of the SM prediction which is in progress. In addition, an analysis of the existing BELLE data would be welcome in order to reduce the error of the measurement. $B \rightarrow X_d \gamma$ is also very interesting for future super-B factories which will be able to measure this decay very precisely.

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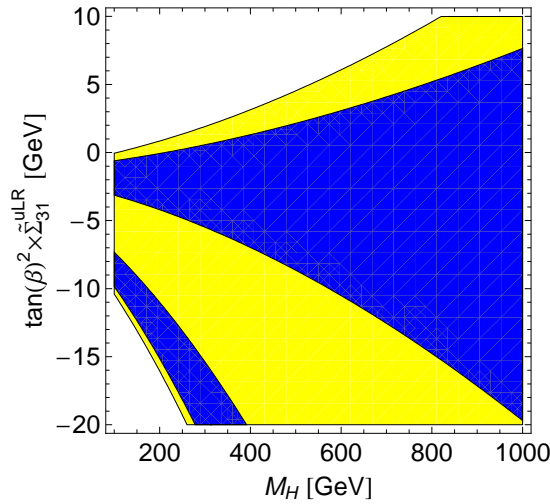


FIG. 4: Allowed regions in the $\tan(\beta)^2 \tilde{\Sigma}_{31}^{u RL} - M_{H+}$ plane in the 2HDM III.

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- [1] K. Nakamura et al. (Particle Data Group), J. Phys. **G37**, 075021 (2010).
 - [2] M. Misiak et al., Phys. Rev. Lett. **98**, 022002 (2007), hep-ph/0609232.
 - [3] M. Misiak and M. Steinhauser, Nucl. Phys. **B764**, 62 (2007), hep-ph/0609241.
 - [4] P. del Amo Sanchez et al. (BABAR), Phys. Rev. **D82**, 051101 (2010), 1005.4087.
 - [5] W. Wang (2011), 1102.1925.
 - [6] B. O'Leary et al. (SuperB) (2010), 1008.1541.
 - [7] T. Abe et al. (Belle II) (2010), 1011.0352.
 - [8] A. Ali and C. Greub, Phys. Lett. **B287**, 191 (1992).
 - [9] A. Ali, H. Asatrian, and C. Greub, Phys. Lett. **B429**, 87 (1998), hep-ph/9803314.
 - [10] T. Hurth and M. Nakao, Ann. Rev. Nucl. Part. Sci. **60**, 645 (2010), 1005.1224.
 - [11] M. Benzke, S. J. Lee, M. Neubert, and G. Paz, JHEP **08**, 099 (2010), 1003.5012.
 - [12] J. Charles et al. (CKMfitter Group), Eur. Phys. J. **C41**, 1 (2005), hep-ph/0406184.
 - [13] M. Ciuchini et al., JHEP **07**, 013 (2001), hep-ph/0012308.
 - [14] T. Hurth, E. Lunghi, and W. Porod, Nucl. Phys. **B704**, 56 (2005), hep-ph/0312260.
 - [15] A. Crivellin and U. Nierste (2009), 0908.4404.
 - [16] A. Crivellin, L. Hofer, and J. Rosiek (2011), 1103.4272.
 - [17] G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B645**, 155 (2002), hep-ph/0207036.
 - [18] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991).
 - [19] J. S. Hagelin, S. Kelley, and T. Tanaka, Nucl. Phys. **B415**, 293 (1994).
 - [20] P. Ko, J.-h. Park, and G. Kramer, Eur. Phys. J. **C25**, 615 (2002), hep-ph/0206297.
 - [21] D. Becirevic et al., Nucl. Phys. **B634**, 105 (2002), hep-ph/0112303.
 - [22] A. J. Buras, M. V. Carlucci, S. Gori, and G. Isidori, JHEP **1010**, 009 (2010), 1005.5310.
 - [23] G. Branco, P. Ferreira, L. Lavoura, M. Rebelo, M. Sher, et al. (2011), * Temporary entry *, 1106.0034.
 - [24] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, Front.Phys. **80**, 1 (2000).
 - [25] A. Crivellin, Phys. Rev. **D83**, 056001 (2011), 1012.4840.
 - [26] P. L. Cho and M. Misiak, Phys. Rev. **D49**, 5894 (1994), hep-ph/9310332.
 - [27] B. Grzadkowski and M. Misiak, Phys. rev. **D78**, 077501 (2008), 0802.1413.
 - [28] A. Crivellin, Phys. Rev. **D81**, 031301 (2010), 0907.2461.
 - [29] A. J. Buras, K. Gemmler, and G. Isidori, Nucl. Phys. **B843**, 107 (2011), 1007.1993.